Teacher Created Materials

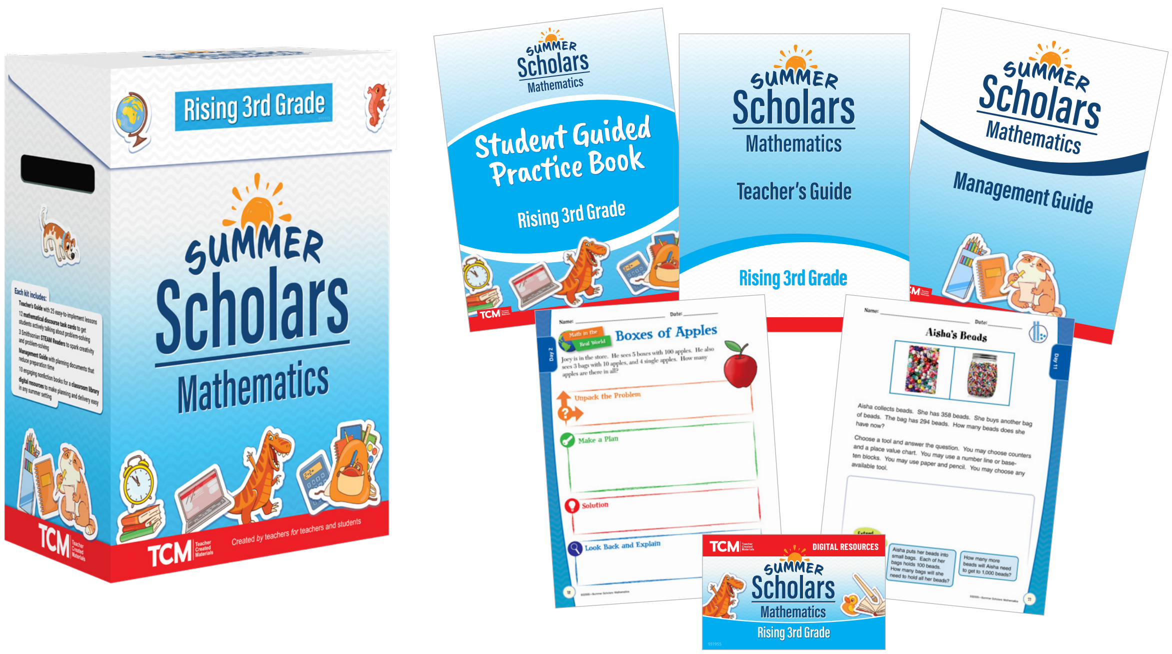
Summer Scholars

Mathematics

A Fresh Approach to Summer School
Based on Respected Research and Literature

Introduction

Summer Scholars: Mathematics was designed specifically for summer learning to help students develop the confidence and academic readiness needed to be successful in the upcoming grade level. The materials and resources focus on key mathematics standards and serve to assist teachers in providing explicit and engaging instruction that inspires students' curiosity and creativity while minimizing preparation and planning time.



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The *Summer Scholars* Logic Model

The Logic Model below demonstrates how *Summer Scholars: Mathematics* is designed to develop mathematical thinkers. Evidence of this is suggested through its resources and activities, which are linked to positive outcomes for students. The goal of this table is to help visualize how implementing *Summer Scholars: Mathematics* can support and contribute to achieving school and district goals.

Problem Statement: There is a need for summer mathematics instruction for rising 1st–6th grade students.					
Outcome/Goal: To help students engage as productive mathematical thinkers and doers					
Theory of Action					
Educators implement evidence-based mathematical strategies.	Rising 1st–6th grade students engage in and utilize mathematical strategies and the engineering design process.	Rising 1st–6th grade students will have increased skills and comfort with mathematical content, practices and processes, and the engineering design process.	Rising 1st–6th grade students will have increased achievement in mathematical content, skills, problem-solving, and the engineering design process.	Rising 1st–6th grade students will be prepared for secondary and post-secondary education success.	Students become lifelong mathematical problem-solvers and critical thinkers.
Logic Model					
Assumptions	**Resources/Inputs**	**Activities**	**Outputs/Metrics**	**Outcomes**	**Impact**
• Students can learn multiple new practices and processes of mathematical thinking and analyze them in math tasks. • Mathematical problem solving is important for school success. • School districts are interested and prepared to incorporate new and different teaching strategies. • Technology is accessible.	• Management Guide with best practices and key research • materials and lesson plans developed through collaboration of experts in the field • protocols to structure students' mathematical discourse • summer-long program offering pacing suggestions for daily lessons spanning 90 or 120 minutes in length • high-interest reading text cards, reader's theater scripts, and classroom library books • audio recordings of mathematical discourse text cards and STEAM challenges • interactive ebooks of STEAM challenges • digital math fluency games • assessments • student-guided activities	• 90- or 120-minute daily lessons • lessons for a variety of instructional settings: whole group, small group, collaborative practice, and independent practice • explicit instruction of key, grade-level appropriate mathematical concepts and skills • explicit instruction in using specific protocols to support mathematical discourse and problem solving • teacher modeling and student practice of mathematical processes • collaborative opportunities to engage with and discuss high-interest math problems • daily opportunities for STEAM exploration and practice of design process • engaging activities and structured practice for students to engage with mathematics, problem solving, and STEAM concepts	• expressive mathematical problem-solving growth • meet or exceed expectations of mathematics standards • completion of lessons, rich problem-solving tasks, and STEAM challenges • robust mathematical discourse • formative and summative assessments • improvement in understanding of grade-level math content and process standards	• knowledge of mathematical practices and processes • deepened overall mathematics awareness and design process • ability to engage in mathematical discourse and mathematical problem solving • increased achievement in mathematics standards • engagement in mathematics and design process	• increased mathematical thinking, discourse, and problem solving among students • development of creative and critical thinking skills • prepared for secondary and post-secondary education success

Guiding Principles

- ➜ High-quality mathematics instruction supports students' development of conceptual understanding and procedural proficiency to build fluency and automaticity.
- ➜ High-quality mathematics instruction prioritizes implementing meaningful, high-cognitive demand tasks, engaging students in using habits of mathematical thinkers, and supporting all learners.
- ➜ Communicating mathematically encompasses speaking, listening, reading, and writing effectively.

These principles are the foundation of *Summer Scholars: Mathematics* and are embedded in every component of the product.

The Need for Intervention

The goal of mathematics education is to provide all students with the ability to use mathematics to improve their own lives, to help them become aware of their responsibilities as citizens, and to help them prepare for their futures. In order to accomplish these goals, state departments of education, school districts, and teachers must set high expectations for all students, and mathematics education needs to be a priority at all levels. *PISA 2012 Assessment and Analytical Framework:*

Mathematics, Reading, Science, Problem Solving and Financial Literacy describes the expectations students are to meet and the experiences they need to have to achieve those expectations. "This conception of mathematical literacy supports the importance of students developing a strong understanding of concepts of pure mathematics and the benefits of being engaged in explorations in the abstract world of mathematics. The construct of mathematical literacy, as defined for PISA, strongly emphasizes the need to develop students' capacity to use mathematics in context, and it is important that they have rich experiences in their mathematics classrooms to accomplish this" (PISA 2013, 25).

Students come to the classroom with different approaches to learning, various levels of mathematical proficiency, language differences, and diverse background knowledge and vocabulary understanding. Teachers must understand the development of mathematics, considering the progression of concepts, strategies, and models that can become powerful forms of representation and tools to think with (Fosnot and Hudson 2010). Student perception may also affect learning. Students approach mathematics instruction with varying levels of readiness. Some students struggle to visualize or develop understanding of abstract concepts. Other students struggle to master mathematical procedures because they do not understand the concept or the rationale for the steps of the procedure. Additionally, students may not possess strategies for attacking an unfamiliar word problem. Whatever the obstacle, it is essential that instruction is designed to meet the mathematical needs of all students before they fail.

"Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena" (PISA 2013, 25).

Research (Foorman and Torgesen 2001) has also found that there should be distinct differences between the type of instruction provided to all students and those identified as needing extra support. Some struggling students make gains during the school year but don't retain that learning over the summer. This is commonly referred to as summer slide or summer learning loss.

What Is Summer Learning Loss?

Summer learning loss refers to the phenomena that students begin a new school year with lower achievement levels than they started with at the beginning of the summer break. Although the extent of learning loss is often greater at higher grade levels (Quinn and Polikoff 2017; Atteberry and McEachin 2016), a seminal meta-analysis of summer learning found that all students loose both reading and mathematics knowledge, with the greatest learning loss occurring in mathematics (Cooper et. al 1996).

Research has also shown that summer learning loss is often greater for low-income students as compared to their more affluent peers (e.g., Augustine et. al 2016; McCombs et. al 2020; Allington 2006), especially in the area of reading (Cooper et. al 1996). One explanation for this comes from Doris Entwisle, Karl Alexander, and Linda Olson's "faucet theory" (2000). The theory posits that during the school year, the "resource faucet" is on for all students, which enables everyone to make learning gains. During the summer, conversely, the flow of resources slows for low-income students but not for higher-income students, who often have access to enrichment opportunities, lots of books and activities at home, and other summer learning opportunities. Sarah Pitcock from the National Summer Learning Association echoes this theory (National Academies of Science, Engineering, and Medicine 2016): "Summer is one of the most inequitable times of year—I believe the most inequitable time of year—across a number of domains…The achievement gap is coming from summer. It is not coming from differences in the way kids learn when they're in school."

The importance of summer programs cannot be overstated. Students make the largest academic gains when they have a high attendance rate, participate in productive use of instructional time, and receive high-quality instruction (McCombs et. al 2020; Quinn and Polikoff 2017; Augustine et. al 2016), but that is not the only benefit. "Summer programs build not only academic skills, but also self-confidence, the ability to focus, and collaborative skills, and these skills can be especially hard to measure" (National Academies of Science, Engineering, and Medicine 2016). Additionally, participation in summer programs has "...demonstrated a number of positive outcomes: increased engagement in school, improved school-day attendance, fewer unexcused absences, fewer disciplinary referrals, improved academic performance, fewer behavior problems, and improved social and emotional competencies" (Naftzger and Newman 2021). Thus, providing access to high-quality summer learning opportunities for as many students as possible should be a priority for districts across the country.

According to the National Summer Learning Association (2020), 9 in 10 teachers report spending at least three weeks at the beginning of the school year re-teaching content from the previous year.

These findings highlight the need for effective and engaging summer intervention to ensure that all students succeed.

Components of Effective Mathematics Intervention

Students' math difficulties are often rooted in challenges with number sense, accuracy in arithmetic combinations, and problem solving (Hanich et al. 2001). Given this, researchers have identified essential elements of effective mathematical interventions such as summer programs. These include explicit, systematic, problem-based instruction in:

1. building conceptual knowledge and procedural understanding
2. developing proficiency in number sense with whole and rational numbers
3. building accuracy and fluency in arithmetic combinations
4. problem-solving

(Gersten et al. 2005)

Building Conceptual Understanding

Learning begins with a concrete representation of a mathematical concept (Cathcart et al. 2000). Research repeatedly shows that students gain greater conceptual understanding and are more successful in demonstrating mastery of concepts when they have had a chance to concretely experience mathematical concepts using manipulatives. In addition, when students use manipulatives, they perform better academically and have more positive attitudes toward mathematics (Leinenbach and Raymond 1996; Jones and Tiller 2017).

Manipulatives, sometimes called math tools or objects to think with, are concrete objects that allow students hands-on experiences while being actively engaged in the learning (Kennedy, Tipps, and Johnson 2008; Horan and Carr 2018). They are often colorful, intriguing materials constructed to illustrate and model mathematical ideas and relationships for students in all grades (Burns and Silbey 2000). Some common manipulatives include:

- ➜ pattern blocks
- ➜ counters
- ➜ base-ten blocks
- ➜ connecting cubes
- ➜ rulers
- ➜ coins
- ➜ algebra tiles

Manipulatives are an effective tool for students to use to build concrete representations because they provide an alternate route to access and develop understanding of mathematics. Manipulatives also support learning by creating physical models that become mental models for concepts and processes (Kennedy, Tipps, and Johnson 2008). Manipulatives help students develop the ability and confidence to see relationships and connections among the domains of mathematics: counting and cardinality, number and operations, base ten, algebraic thinking, measurement and data, geometry, and statistics and probability.

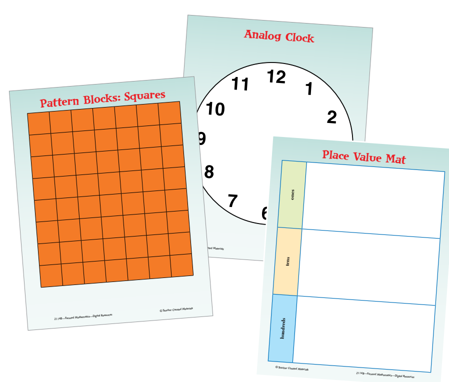
As students build their conceptual understanding using concrete objects (manipulatives), instruction moves to the representational phase (drawings, tallies, dots, etc.) and finally the abstract phase, where numbers and symbols are used to represent the concept (Witzel 2005). This instructional sequence is commonly referred to as concrete-representation-abstract, or CRA.

Here is a step-by-step example of how a teacher can use manipulatives to help students transition from the concrete to the abstract:

1. Explain the role of manipulatives, how they connect to an overall mathematical concept, and the expectations for student use.
2. Give students practice in using the manipulatives to explore the mathematical concept.
3. Model the mathematical concept with pictures. Make connections between the manipulatives and the pictures.
4. Give students practice in using pictures (as a substitution for the manipulatives) to explore the mathematical concept.
5. Teach the abstract qualities of the mathematical concept. Make connections between the pictures and the equations or formulas.
6. Provide ample opportunities to practice problem-solving procedures without pictures or manipulatives.
7. Return to manipulative use when needed, repeating this entire process to move students to abstract thinking and problem solving.

Research to Practice

In *Summer Scholars: Mathematics*, manipulatives are used during instruction to help make abstract ideas more concrete. Digital copies of the manipulatives can be found in the digital resources. Lessons using manipulatives include the filenames in the materials lists.



Developing Math Fluency

Math fluency is the idea that a student must be both accurate and fast when solving basic math facts (addition, subtraction, multiplication, and division) (Lin and Kubina 2005). Research shows that students with strong math fluency and automaticity (the ability to recall a correct answer to a math fact immediately) are more likely to have sustained success in mathematics (e.g., Stickney et al. 2012; Woodward 2006). Thus, it should not be surprising that research also shows that students with difficulties in mathematics often lack proficiency in computational skills (Baker and Cuevas 2018; Bryant et al. 2008; Gersten, Jordan, and Flojo 2005; Calhoon et al. 2016).

There is an emphasis in national mathematics standards for students to be able to solve math problems accurately and efficiently (i.e., to demonstrate fluency and automaticity). While fluency with key mathematics skills, such as recall of basic facts, is certainly expected, it is important to realize that conceptual understanding is the basis for developing fluency and automaticity, especially with students who struggle and cannot depend on rote memorization. When a student understands combinations of tens, developed through many experiences using a ten frame, they can extend that understanding to composing and decomposing numbers to learn difficult addition facts. For example, the student can think about 8 + 5 as taking 2 from the 5 and adding it to the 8 so the fact now becomes 10 + 3, which equals 13.

Students who struggle with mathematics need many opportunities and models to build this foundational understanding before they can simply memorize their facts. It cannot be overemphasized that intervention students need more experiences than what is provided in a usual mathematics class in order to develop the conceptual understanding needed to reach a level of fluency. Developing fluency begins with conceptual understanding, strategy development through the use of appropriate models and tools, and explicitly helping students make connections between those models and basic facts. The earlier such interventions take place, the greater chance for success in not only helping students become fluent with facts, but also extending their foundational understanding to more complex whole-number operation concepts.

Research to Practice

Summer Scholars: Mathematics incorporates a balanced approach to develop both conceptual understanding and mathematical fluency.

- In the Warm-Up activities, students activate prior conceptual knowledge, review prerequisite skills, and reinforce numeracy skills.
- In the Whole-Group Lesson and Differentiated Instruction, students construct understanding of mathematical concepts and then apply that understanding to building fluency and automaticity with mathematical procedures.
- Students further practice and reinforce key skills through engaging math fluency games. There are games provided in the digital resources in each level of *Summer Scholars: Mathematics.*

Problem Solving Using Rich Mathematical Tasks

Mathematical problem solving is a challenging aspect of instruction for many mathematics educators. However, it is essential to create an engaging learning environment in which students' mathematical understanding grows through systematic, explicit modeling, with multiple opportunities for guided and independent problem solving.

Problem solving is a key reason for learning mathematics. It is through problem solving that we can look at a situation, analyze it, and determine possible solution paths and reasonable solutions (Guzman Gurat 2018). It is problem solving that makes mathematics meaningful in our daily lives.

Unfortunately, students who are identified as needing intervention are often limited to routine problems that involve low cognitive demand, which simply provide practice for the mathematics (e.g., computational procedure) they just learned. Struggling students should have opportunities to solve challenging, non-routine problems. The role of the teacher is to scaffold high cognitive-demand problems so that struggling students have entry to the problem. For example, linkages to more complex tasks may need to be more explicit for students who struggle. To avoid giving intervention students opportunities to solve rich problems is to shortchange their mathematical experiences. Since evidence shows that problem solving using academically challenging tasks with a focus on reasoning offers the greatest learning opportunities for students, it is critical that all students have access to these types of tasks (Smith and Stein 1998).

Another way that teachers can scaffold the problem-solving process for students is through learning and applying a protocol for using problem-solving strategies. The work of George Pólya [1945] (2015) helps provide a framework for students as they tackle problems. Students need explicit instruction and practice using the framework. The most used steps include the following: understand the problem, devise a plan, carry out the plan, and look back.

1. Understand the Problem

The first step to understanding the problem is to have students (or the teacher, if necessary) read the problem either silently or aloud. For many students, putting the problem in their own words helps them make sense of the information and the question. Asking the following questions helps students focus on the critical parts of a problem (Gojak 2011):

- ➜ *What do you know?* Discuss the information in the problem.
- ➜ *What do you want to find out?* Focus on the question.
- ➜ *What information will help you answer the question? What information is extra?*
- ➜ *Do you need any other information to find a solution?* This question can help students identify steps in multistep problems.
- ➜ *What might be a reasonable answer to this problem*? The point here is not to answer the question but to lead students to make sense of the problem situation and the solution.

2. Devise a Plan

Once students understand the problem context, they can begin to associate the question and the information given in terms of mathematical ideas or operations. In some problems, the plan may be directly related to the meaning of an operation. In others, specific problem-solving strategies will be helpful in planning a path to the solution. All students should have explicit instruction and multiple opportunities to use problem-solving strategies. Intervention students need additional scaffolding in using the strategies, especially when more than one strategy might be needed.

3. Carry Out the Plan

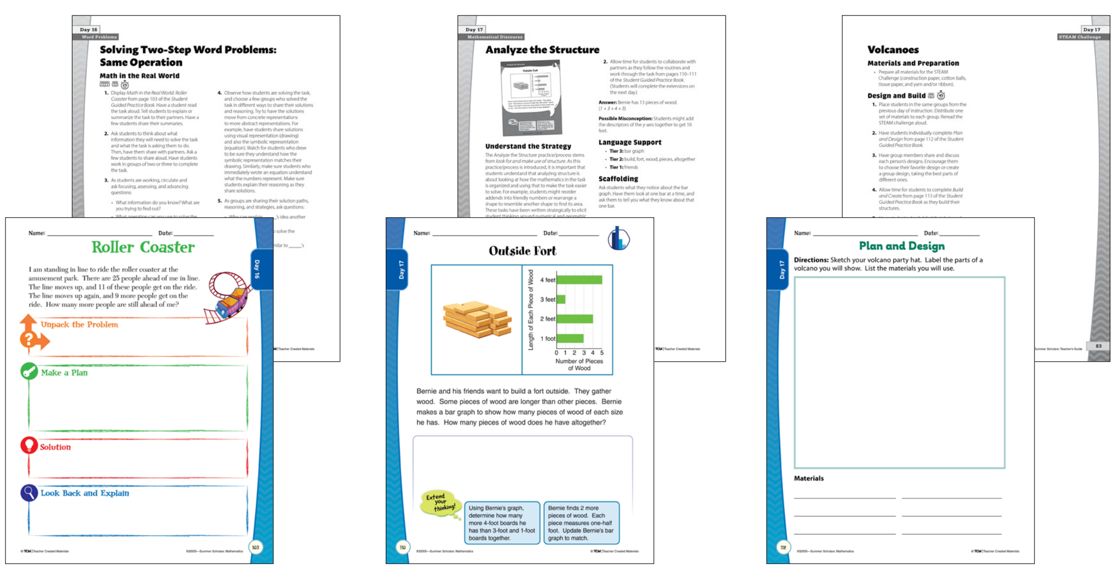
Students who have difficulty solving problems often skip the first two steps and jump right to working on the problem. This is usually where they get stuck. Students must complete the first two steps before attempting to solve the problem. Only when they complete the first two steps are they ready to begin the actual work of solving the problem. An important part of this step is for students to check their thinking. Are they headed in the right direction or down the wrong path? Is the strategy they selected working, or do they need to try something else? Teachers should ask questions to help students become more independent problem solvers. The expectation for students' written work should be that it is organized and clear. This not only lessens the possibility of getting lost in the solution process, but also helps students communicate their mathematical thinking and representations. It is through clear communication and representations that students are more likely to make connections among mathematical ideas and real-life applications.

4. Look Back

Too often, students think that the goal of mathematics is to get to the answer, and then the thinking stops. By looking back, students can think about their work and the reasonableness of their solutions given the constraints of the problem. Additionally, discussing their thinking with one another helps them make sense of mathematical ideas and relationships. It is critical for the teacher to ask questions and lead discussions—especially for students who struggled with the problem.

Research to Practice

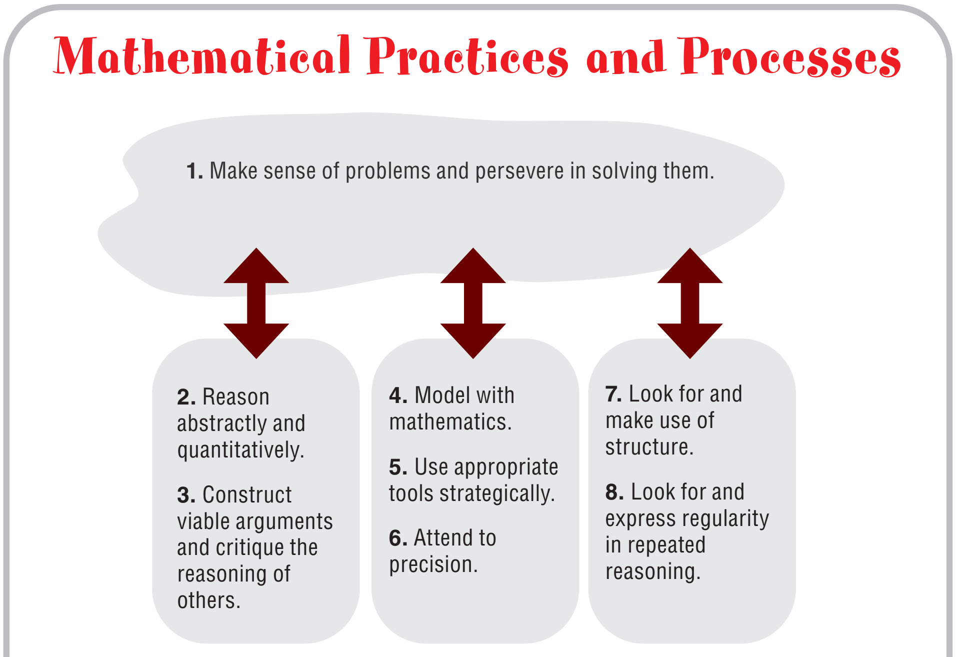
Summer Scholars: Mathematics also helps teachers scaffold problem-solving opportunities so that students have entry to rich problems and mathematical tasks. Math in the Real World tasks walk students through the problem-solving process step by step. The Mathematical Discourse Task Card routines incorporate the problem-solving steps to guide student thinking and discussions. The STEAM challenges incorporate these steps and help lead students through the processes of inquiry and problem solving. Embedded into each of these rich real-world tasks are many opportunities for teachers to make important points about mathematics content and mathematical thinking.



Integrating Math Practices/Processes

In 2000, the National Council of Teachers of Mathematics (NCTM) released the "Principles and Standards for School Mathematics: An Overview," which deepened the understanding that mathematics is a combination of content and process, encouraging the expectation of standards-based teaching (NCTM 2000). Following its release, a project sponsored by the National Science Foundation and the U.S. Department of Education published "Adding It Up: Helping Children Learn Mathematics" (National Research Council 2001). This publication introduced the five strands of mathematical proficiency. The intent of the report was to ensure that students become proficient in math content and processes. This laid the groundwork for agencies such as the National Governors Association Center for Best Practices, the Council of Chief State School Officers, and state education departments to develop mathematical content and practice/process standards that focus on the conceptual and procedural understanding children must have to develop mathematical proficiency (2014). The standards are designed as progressions, each level building upon the next. The documents are interconnected works that describe the expertise that all mathematics educators should develop in their students to build their proficiencies in mathematical understanding, reasoning, and application.

Mathematical thinking is the key to mathematical literacy. "Mathematical thinking is a whole way of looking at things, of stripping them down to their numerical, structural, or logical essentials, and of analyzing the underlying patterns. Moreover, it involves adopting the identity of a mathematical thinker" (Devlin 2012). To develop these habits of mind, states have set forth specific mathematical processes and practices that students must master. Students are to build proficiency with these processes and practices as they master the content standards for their grade levels. As students develop proficiency with the process and practice standards, they will be more successful problem solvers, use mathematics effectively and efficiently in daily life, and become college- and career-ready (Texas Education Agency 2012). The figure below details the eight Mathematical Practices and Processes and illustrates how they are interconnected and work together.



Adapted from Bill McCallum blog at University of Arizona

Figure 2—Mathematical Practices/Processes used with permission. From *Engage in the Mathematical Practices: Strategies to Build Numeracy and Literacy with K–5 Learners* by Kit Norris and Sarah Schuhl. Copyright 2016 by Solution Tree Press, 555 North Morton Street, Bloomington, IN 47404, 800.733.6786, SolutionTree.com. All rights reserved.

Ultimately, teachers must prepare students for the future. This means training students to be mathematical thinkers by utilizing skills that are important to their future success. These documents, and the state standards that have evolved from them, are designed to close the opportunity gap and provide all students equal opportunity to achieve mathematical literacy.

Research to Practice

Classrooms that support twenty-first century learners look like collaborative spaces, not assembly lines, and strategically integrate opportunities for students to practice the Mathematical Practices and Processes. *Summer Scholars: Mathematics* provides opportunities for students to work together to make sense of mathematical tasks while simultaneously thinking critically and creatively about their problem-solving processes.

The Importance of Vocabulary Instruction

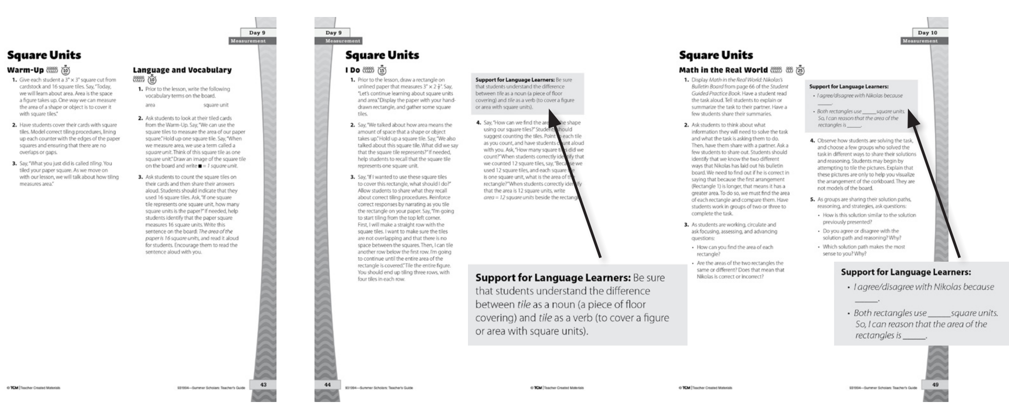
In mathematics, vocabulary is highly specialized. These words are often not encountered in everyday life. Therefore, all students need an explicit introduction and explanation of these vocabulary words in order to apply them to their understanding of mathematical concepts.

Research has consistently found a deep connection between vocabulary knowledge, reading comprehension, and academic success (Baumann, Kame'enui, and Ash 2003). Michael Kamil and Elfrieda Hiebert describe vocabulary as a bridge between the "word-level processes of phonics and the cognitive processes of comprehension" (2005, 4). This is a useful way to visualize the importance of vocabulary for students who struggle with mathematics.

Mathematical language can also hinder student learning, causing students with math difficulties to focus on terms and definitions instead of the mathematical relationships involved. By using correct terminology within the context, students can integrate the words more naturally into their vocabulary (Fosnot and Hudson 2010). Students who are struggling with mathematical concepts or students who have not shown mastery of the vocabulary also need structured lessons to focus attention on the content words.

Research to Practice

In *Summer Scholars: Mathematics*, mathematical language and vocabulary activities are provided at the start of every lesson, and additional practice is included in the Mathematical Discourse Task Cards. There are also language frames to support vocabulary development specifically written for English learners; however, they can be used to support the math content/language learning of all students.



Promoting Math Discourse

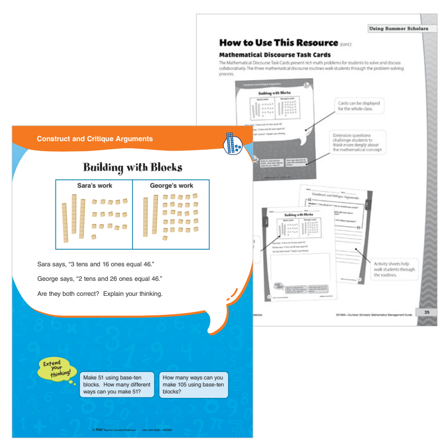
Research has shown that discourse is fundamental to mathematics learning. The idea that a classroom of mathematics learners should be led by and centered on their justifications, reasonings, and discussions is the backbone of mathematical discourse. Classrooms today must prepare students for the twenty-first-century workforce, which is a collaborative and innovative space where people must be able to communicate their ideas clearly and concisely, listen carefully to others' thinking and strategies, determine how to compromise and move forward collectively, and continue discussions so that all perspectives are valued.

For students to be prepared for this type of collaborative work environment, they must learn how to engage appropriately in discussions. This should start as early as kindergarten. Establishing a mathematics classroom focused on collaborative problem solving enables students to develop the language skills they need to communicate effectively both within and beyond mathematics. Such a mathematics classroom places the responsibility for learning squarely on the shoulders of the students.

In addition to collaboration, math discourse is also about communicating mathematical ideas effectively. Communicating mathematically means that students are able to speak, listen, read, and write in order to share ideas, clarify understanding, and build meaning for ideas (National Council of Teachers of Mathematics 2000). The use of structured discourse centered around problem-solving routines helps students slow down, comprehend the situation, and communicate about the math. According to Grace Kelemanik, Amy Lucenta, and Susan Creighton, "Instructional routines are meant to be repeated, and this repetition makes them very effective vehicles for developing mathematical practices" (2016, 4). Antonia Cameron, Patricia Gallahue, and Danielle Iacoviello (2020) also emphasize the importance of mathematical routines, specifically routines that have predictable patterns and are meant for students to remember. By following these routines, students become successful problem solvers and can more effectively communicate their ideas.

Research to Practice

The Mathematical Discourse Task Cards focus on mathematical discourse. The cards are centered on three instructional routines. These routines create structured discourse opportunities and help students optimize their outputs, both in problem solving and in use of language. The *Management Guide* provides extensive information on how to successfully introduce and implement the routines.



The Importance of STEAM Education

STEM has become a common educational acronym over the past decade, standing for *science, technology, engineering,* and *math.* Creativity is another essential component for innovation, unaccounted for with the traditional acronym. Thus, the need for creative thinkers helped to launch the STEAM movement. “The *A* is where STEAM and making intersect. It is at this intersection where student engagement soars” (Maslyk 2016,10). Blending arts principles with STEM disciplines prepares students to be problem solvers, creative collaborators, and thoughtful risk-takers. Even students who don’t choose a career in a STEM or STEAM field will benefit because these skills can be translated into almost any career. Rodger W. Bybee (2013, 64) summarizes what is expected of students as they join the workforce: “As literate adults, individuals should be competent to understand STEM-related global issues; recognize scientific from other nonscientific explanations; make reasonable arguments based on evidence; and, very important, fulfill their civic duties at the local, national, and global levels.”

Likewise, STEAM helps students understand how concepts are connected as they gain proficiency in the Four Cs: creativity, collaboration, critical thinking, and communication.

The content and practices of STEAM education are strong components of a balanced instructional approach, ensuring students are college- and career-ready. The application of STEAM practices in the classroom offers teachers opportunities to challenge students to apply new knowledge. Students of all ages can design and build structures, improve existing products, and test innovative solutions to real-world problems. STEAM instruction can be as simple as using recycled materials to design a solar cooker for hot dogs and as challenging as designing a solution to provide clean water to developing countries. The possibilities are endless.

STEAM is an integrated way of preparing students for the twenty-first century world. It places an emphasis on understanding science and mathematics while learning engineering skills. By including art, STEAM recognizes that the creative aspect of any project is integral to good design—whether designing an experiment or an object.

Research to Practice

In each level of *Summer Scholars: Mathematics*, there are five STEAM challenges that guide students through the engineering design process to solve a problem. Each challenge is completed over five days to give students ample time to investigate, test, and retest their ideas. In addition to meeting specific criteria, students are also challenged to improve their work over the five days and use creativity, collaboration, critical thinking, and communication skills to find solutions.

Science

Any project or advancement builds on prior science knowledge. Science focuses on learning and applying specific content, cross-cutting concepts, and scientific practices that are relevant to the topic or project.

Technology

This is what results from the application of scientific knowledge and engineering. It is something that is created to solve a problem or meet a need. Some people also include the use of technology in this category. That is tools used by scientists and engineers to solve problems. In addition to computers and robots, technology can include nets used by marine biologists, anemometers used by meteorologists, computer software used by mathematicians, and so on.

Engineering

This is the application of scientific knowledge to meet a need, solve a problem, or address phenomena. For example, engineers design bridges to withstand huge loads. Engineering is also used to understand phenomena, such as designing a way to test a hypothesis. When problems arise, such as earthquakes or rising sea levels, engineering is required to design solutions to the problems. On a smaller scale, a homeowner might want to find a solution to their basement flooding.

Art

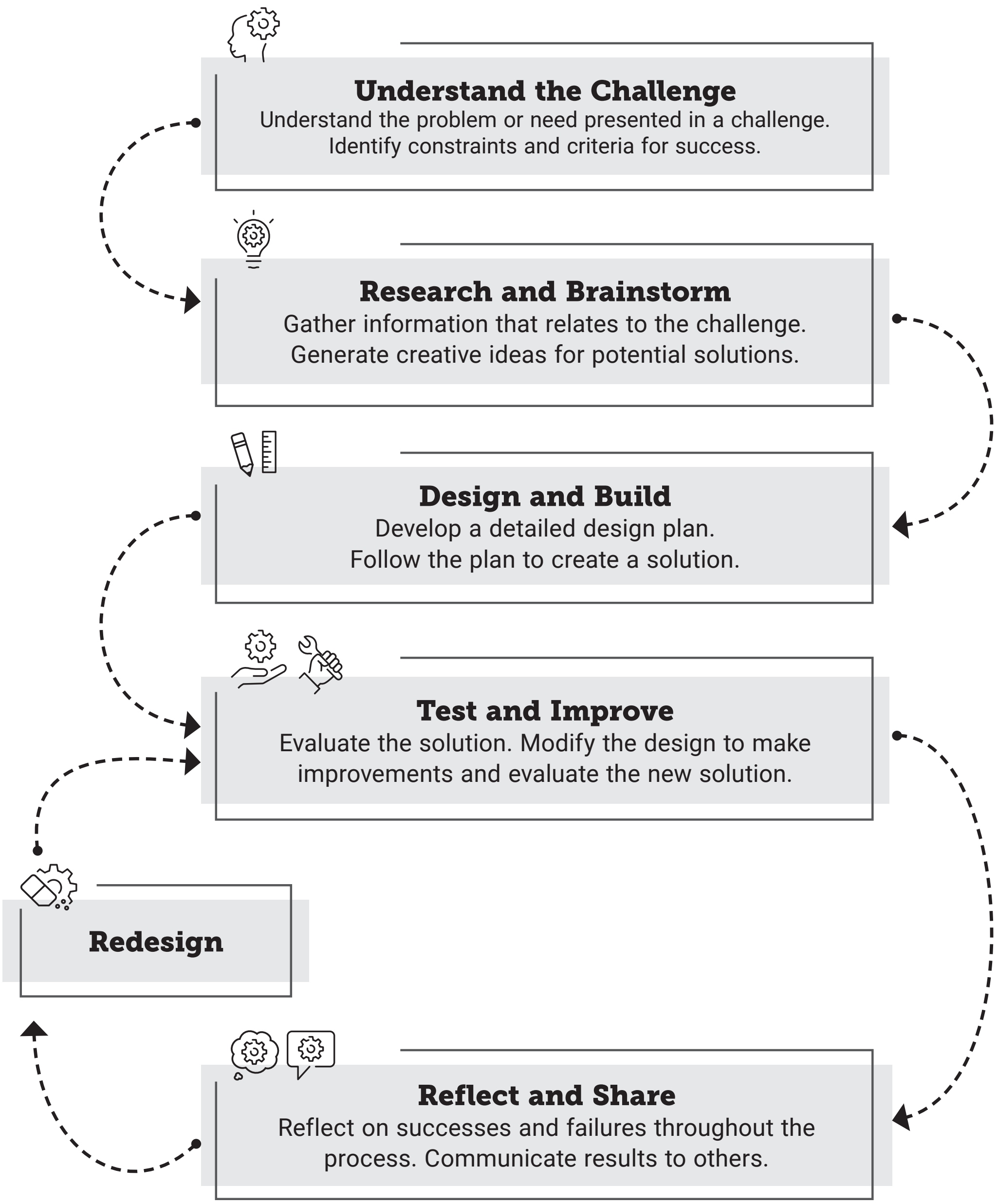
In this context, art equals creativity and creative problem solving. For example, someone might want to test a hypothesis but be stumped as to how to set up the experiment. Perhaps you have a valuable painting. You think there is another valuable image below the first layer of paint on the canvas. You do not want to destroy the painting on top. A creative solution is needed. Art can also include a creative or beautiful design that solves a problem. For example, the Golden Gate Bridge is considered both an engineering marvel and a work of art.

Mathematics

This is the application of mathematics to real-world problems. Often, this includes data analysis—such as collecting data, graphing it, analyzing the data, and then communicating that analysis. It may also include taking mathematical measurements in the pursuit of an answer. The idea is not to learn new math but to apply it; however, some mathematics may need to be learned to solve a specific problem. Isaac Newton, for example, is famous for inventing calculus to help him solve problems in understanding gravity and motion.

The Engineering Design Process

The most essential component of STEAM education is the engineering design process. This process is an articulated approach to problem solving, in which students are guided through the iterative process of solving problems and refining solutions to achieve best possible outcomes. “It is important to point out that these components do not always follow a set order, any more than do the ‘steps’ of scientific inquiry. At any stage, a problem solver can redefine the problem or generate new solutions to replace an idea that just isn’t working out” (NGSS Lead States 2013, 2). Each lesson in this series presents students with a design challenge that guides them through the engineering design process to solve a problem.



Using Technology to Support Instruction

It is important to integrate technology into purposeful instructional objectives. Technological tools, when used appropriately, support sound instructional practices. For example, research shows that high-quality math apps and other technology can have a positive effect on student achievement (Shujaa 2022).

Integration of technology is not a substitute for effective teaching practice, but rather can be used to enhance proven strategies. Devices, software, and learning management systems are effective tools to scaffold learning, allow for increased independent learning, and provide multiple means for students to interact with texts and demonstrate understanding.

Using Interactive Texts

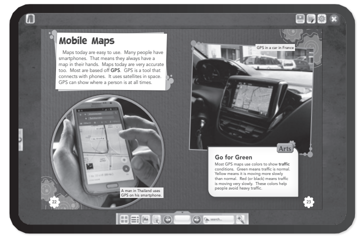
Interactive texts offer educators the unique opportunity to integrate technology into their curriculum for reading or content-area literacy instruction. Interactive texts guide students toward independent reading while exploring core concepts. Teachers can determine whether to use interactive texts in place of the print versions of books or to use them as a supplement. The implementation of interactive texts will depend on the electronic resources available to both teachers and students (e.g., the availability of a projector or the number of student devices) and the method of use (e.g., whole-class, small-group, or individual learning opportunities).

Interactive texts can enhance student learning in a variety of instructional settings, support English language acquisition, and further content and literacy learning. They include annotation tools, embedded audio recordings to model language and intonation, and recording tools for fluency practice. Each interactive text includes multiple interactive activities that can be used to strengthen and support student acquisition of essential concepts.

Using interactive texts in conjunction with printed texts allows teachers to demonstrate and model reading skills and strategies or teach content using the interactive features while students read and follow along in printed texts.

Research to Practice

As part of the digital resources, *Summer Scholars: Mathematics* includes Interactiv-eBooks of each of the Smithsonian STEAM Readers. These engaging digital versions of the text help students connect with the content in a variety of ways. Interactiv-eBooks also have a variety of features that build literacy and engage readers:



- Text-to-speech highlighting supports reading fluency.
- Professional audio recordings promote fluency and vocabulary development.
- Interactive activities enrich the reading experience.
- Annotation tools offer opportunities to interact with the text and build key comprehension skills.
- Writing activities offer opportunities to make reading-writing connections.
- Videos add real-world context to the topics in the book.

Interactiv-eBooks have many features that build content knowledge:

- Digital activities can be used to introduce, reinforce, or assess learning.
- Easy-to-use tools give students power to increase comprehension and master vocabulary.
- Interactiv-eBooks allow for comprehension of content from diverse media.

Gamification

Games are a proven source of motivation. They are an engaging way for students to develop, maintain, and reinforce mastery of essential mathematical concepts and processes. Games eliminate the tedium of most mathematics skill drills. The article "Gamification in Education: What, How, Why Bother?" by Lee and Hammer (2011) discusses the benefits and learning potential of using games in the classroom. Citing a variety of research (e.g., Locke 1991; Bandura 1986; Gee 2008; Locke and Latham 1990), the authors discuss various advantages, including the motivation provided by specific, somewhat difficult, immediate goals. They also discuss how games support motivation and engagement by providing many paths to success, giving students the opportunity to choose smaller goals within the larger task. Attitudes are also an important part of success. Students who are engaged and feel good about a subject and their ability to do well in it will be motivated to learn. It is important to provide a positive learning environment where students are under minimal stress; meaning and understanding (rather than rote memorization) are emphasized; real-world concepts are related; and students work in well-organized groups. The use of learning games can be a key aspect in creating a positive learning environment during the summer.

Research to Practice

Each level of *Summer Scholars: Mathematics* includes three Digital Math Fluency Games that attend to the key math fluency skills addressed in the kit. The Digital Math Fluency Games can be used in a variety of instructional settings to guide students toward independent skill application while engaging them in a fully interactive experience.

Supporting All Learners

Since the National Assessment of Educational Progress (NAEP) began tracking the income achievement gap back in 2003, the gap has essentially remained statistically unchanged. The gap persists between white students and students of color, as well as between native English speakers and English learners (U.S. Department of Education 2022). Therefore, summer intervention programs must support the mathematics development of all students in order to support academic growth.

To become mathematicians and problem solvers, students must spend time practicing. Sometimes, teachers skip giving word problems to students who are not native English speakers, who may not find a task relevant, or who struggle mathematically. This is problematic because it does not build the skills needed to become efficient and effective problem solvers. Avoidance does not support all learners. Other times, teachers lessen the language in a task to reduce the cognitive load, but that inadvertently takes away the opportunity for students to be exposed to rich language. *Summer Scholars: Mathematics* offers supports for all learners throughout the lesson plans so that teachers can feel confident about offering challenging problems, tasks, and instruction to all students.

Another aspect of supporting all learners is differentiating instruction to meet learners' needs. While teachers should allow students to productively struggle and persevere through challenges, at times, students' thinking may need to be guided by providing additional support. The use of differentiation will help students engage more deeply in their learning, which allows them to build on their prior knowledge and conceptual understanding and transfer that knowledge to build deeper understanding of the curriculum (Tomlinson 2017; Marks, Woolcott, and Markopolous 2021). Therefore, scaffolding, reteaching, extending the learning opportunities, and addressing possible misconceptions should be considered during instruction.

Language Support

Intervention for English learners should engage students in meaningful activities as well as cognitively demanding content, while scaffolding the content to ensure that students will learn successfully (Diaz-Rico and Weed 2002). Scaffolding in lessons, modeling effective problem-solving strategies, using concrete manipulatives and visual representations, and vocabulary/concept development instruction are vital for

English learners. Additionally, showing images and videos and using gestures can be helpful to make sure students understand a concept or a language-dense text or problem.

Given that mathematics is a language, it is also critical to provide students with proper vocabulary and language development opportunities. This will help students better understand the concepts and apply the mathematics they know. However, because of the range of language proficiency levels, school experiences, and home language supports in English, meeting the needs of English learners who struggle with math can be more complex than meeting the needs of native English speakers who struggle with math.

In addition to direct, explicit instruction, interactive teaching that uses techniques such as modeling and guided practice helps students master requisite skills more effectively (Goldenberg 2010). Therefore, it is important to preteach the words that are critical to understanding a concept or a mathematical text so that students are provided with a variety of ways to learn, remember, and use the words and concepts (Echevarria, Vogt, and Short 2004). It is not enough to simply expose English learners to language-rich classrooms; they need "intensive instruction of academic vocabulary, and related grammatical knowledge must be carefully orchestrated across the subject areas for language minority students to attain rigorous content standards" (Feldman and Kinsella 2005).

In additional to explicit vocabulary support, student talk should be prioritized and heavily scaffolded to ensure comprehension and to optimize output. Talk gives students opportunities to try out language, make errors, self-monitor, and fix their language to communicate effectively. As with any new skill the body undertakes, there is a great need to practice and to have multiple and varied opportunities to use the skill. According to research, students in language-rich environments who solve real-world math problems and use multiple modes of communication develop better English language skills than students in classrooms that do not provide opportunities to experiment with language (Chval and Chavez 2012).

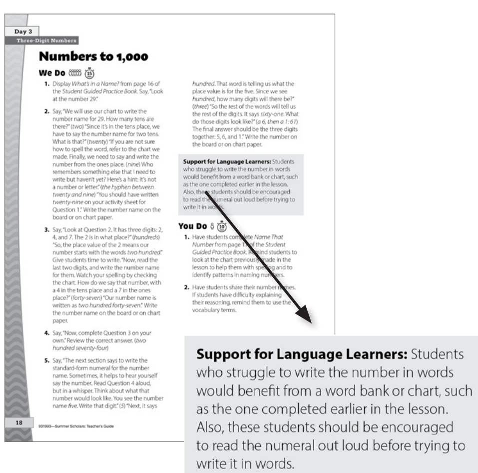
As important as talk is for English learners to develop fluency, it should be coupled with language interactions. Talk-alone will not provide students with the oral rehearsals that demonstrate the complexities of becoming fluent in a language. Students need to use language for authentic purposes, where they exchange language with others through oral discussions. Talking with others gives students immediate feedback to know whether their ideas are being understood and their use of language was effective. Without this exchange, students miss out on valuable feedback to develop their self-monitoring skills (Swain 1985). Discussions further offer students opportunities to learn from one another, both in ideas and language. Hearing other language models and gaining greater exposure to how people think and how those ideas can be translated into comprehensible output further the language development process.

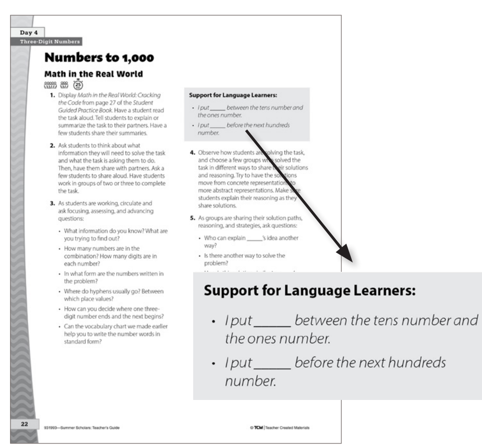
The exchange of language exposes students to different discourse patterns. When English learners engage in discussions with others, they are developing what Susan Ervin-Tripp (1991) refers to as linguistic capital for forms of language, such as negotiating, persuading, questioning, and encouraging. What must be coupled with these language exchanges and authentic oral discourse is access to language supports that students can use to successfully engage in the discussion. William Saunders, Claude Goldenberg, and David Marcelletti (2013) found that "communication and meaning should be used to motivate and facilitate second-language learners' acquisition and use of targeted language functions and forms" (21). They emphasize that students should be encouraged to engage in meaningful exchanges but need ongoing explicit support to do so.

Research to Practice

Throughout the lessons plans, specific suggestions are provided to support the needs of English learners. Those suggestions are strategically placed to support the unique content of each lesson and recommend research-based strategies such as: the use of language frames and visuals, the explanation of multiple-meaning words, or calling attention to specific language or vocabulary to support instruction.

The use of the digital tools, such as the Interactiv-eBooks and audio recordings of the texts, can also be used to scaffold instruction or provide opportunities to build conceptual fluency.





The Importance of Assessment

Assessment is an integral part of good instruction and should be conducted regularly, especially in an intervention or summer learning setting. "Assessment is the collection of data such as test scores and informal records to measure student achievement, and evaluation is the interpretation and the analysis of this data. Evaluating student progress is important because it enables the teacher to discover each student's strengths and weaknesses, to plan instruction accordingly, to communicate student progress to parents, and to evaluate the effectiveness of teaching strategies" (Burns, Roe, and Ross 1999).

Many different types of assessment tools are available in today's schools, including, but not limited to, standardized tests, anecdotal records, informal reading inventories, portfolios, and summative assessments. While each type of assessment serves a different purpose, the true purpose of assessment is to help educators make good decisions about the kind of instruction students need in the classroom (Gresham and Little 2012).

Types of Assessment

"Monitoring and record keeping provide the critical information needed to make decisions about the student's future instruction" (National Center for Learning Disabilities 2006, 5). The ability to properly diagnose and monitor students' progress is imperative in mathematics intervention programs. Teachers must be able to provide instruction that is tailored to the needs of each student. "Teachers can build in many opportunities to assess how students are learning, and then use this information to make beneficial changes in instruction" (Boston 2002).

Diagnostic/Pre-Assessments

Diagnostic or pre-assessments are usually administered prior to the start of program or unit of study to get an idea of students' current knowledge base and levels of understanding. The results provide a baseline that can be used to gauge progress periodically or measure against overall academic growth at the end.

Formative Assessments

Formative assessments may be used to determine the point-in-time status of students' understandings and make decisions about next instructional steps. Marilyn

Burns (2005) shares that formative assessment gives information to teachers about what students understand and shows possible misconceptions. Strategies such as utilizing open questions/tasks as well as observing, listening, and reviewing student work should all be key components in formative assessment in the mathematics classroom, with the goal of using this information to guide instruction. This can also help teachers plan to maximize instructional time.

“When teachers know how students are progressing and where they are having trouble, they can use this information to make necessary adjustments to instructional approaches or offer more opportunities for practice. These activities can lead to improved student success” (Boston 2002). Progress-monitoring assessments can be administered in both formal and informal ways. Teachers use formative assessments to help them make good decisions about the kind of instruction their students need (Honig et al. 2000). These assessments are usually conducted as an ongoing process.

Research to Practice

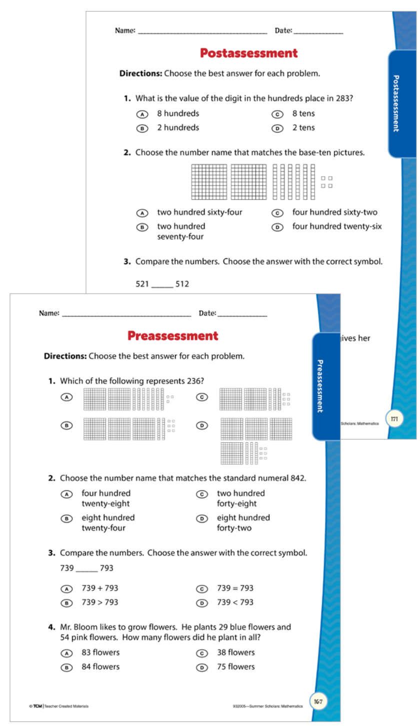
Each level of *Summer Scholars: Mathematics* provides teachers with numerous opportunities for assessment.

Diagnostic/Preassessment: The preassessment can be used as a baseline of student academic readiness. The assessment is found in the *Student Guided Practice Book* as well as in Google Forms™ and Microsoft Documents®.

Formative Assessment: The activity pages from the *Student Guided Practice Book* can be used as a formative assessment.

Progress Monitoring: Quick Check activities allow teachers to see which students need reteaching every other day of instruction. The lessons then have students move through differentiated rotations based on the results of the Quick Check.

Summative/Postassessment: The postassessment provides a record of student growth and academic achievement as a result of using the program. The assessment is found in the *Student Guided Practice Book* as well as in Google Forms™ and Microsoft Documents®.


Name: Date:

Postassessment

Directions: Choose the best answer for each problem.

1. What is the value of the digit in the hundreds place in 283?
   - A 8 hundreds
   - B 2 hundreds
   - C 8 tens
   - D 2 tens
2. Choose the number name that matches the base-ten pictures.
   - A two hundred sixty-four
   - B two hundred seventy-four
   - C four hundred sixty-two
   - D four hundred twenty-six
3. Compare the numbers. Choose the answer with the correct symbol.

   521 ____ 512

Name: Date:

Preassessment

Directions: Choose the best answer for each problem.

1. Which of the following represents 236?
2. Choose the number name that matches the standard numeral 842.
   - A four hundred twenty-eight
   - B eight hundred twenty-four
   - C two hundred forty-eight
   - D eight hundred forty-two
3. Compare the numbers. Choose the answer with the correct symbol.

   739 ____ 793
   - A 739 + 793
   - B 739 > 793
   - C 739 = 793
   - D 739 < 793
4. Mr. Bloom likes to grow flowers. He plants 29 blue flowers and 54 pink flowers. How many flowers did he plant in all?
   - A 83 flowers
   - B 84 flowers
   - C 38 flowers
   - D 75 flowers

Summative Assessments

According to Peter Airasian, the purpose of summative assessment is "to judge the success of a process at its completion." It provides students the opportunity to demonstrate their mastery of concepts taught, which in turn also helps guide teachers' instructional planning. This type of assessment shows growth over time and helps set instructional goals to address students' needs. It also helps to determine how to re-evaluate earlier strategies or steps that will, therefore, influence what follows on a student's academic or instructional path (Airasian 2005).

Conclusion

This is the age of science, technology, and mathematics. To have a mathematically literate society, the population needs to have an understanding of and proficiency with mathematics concepts and procedures, as well as the ability to apply that knowledge, use it to develop models, and apply those models to new situations. The goal of mathematics education is to provide all students with the ability to use mathematics to improve their own lives, to help them become aware of their responsibilities as citizens, and to help them prepare for their futures.

"The summer months present youth with opportunities for academic, physical, and social and emotional growth, but also the possibility of stagnation or decline." (National Academies of Sciences, Engineering and Medicine 2019). According to research studied by the Brookings Institute, on average, students' achievement scores declined over summer by one month's worth of school-year learning for students who did not attend a summer learning program (Quinn and Polikoff 2017). Summer learning programs using resources such as *Summer Scholars: Mathematics* can help address the learning needs of students during the summer months to help prevent learning loss and instead build academic understanding and growth as they head into the next school year.

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