

Building Mathematical Comprehension



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The Seven Comprehension Strategies


In compiling the strategies good readers use, Keene and Zimmermann (2007) drew six from the research of Pearson et al (1992) and added a seventh strategy—*monitoring meaning* (figure 1.2).

Fig. 1.2. Comprehension Strategies

1. Making connections—using schema and building background knowledge
2. Asking questions—generating questions before, during, and after reading to clarify understanding
3. Visualizing—using sensory and emotional images to deepen and expand meaning
4. Making inferences—using background knowledge with new information to predict, conclude, make judgments, or interpret
5. Determining importance—deciding what information is significant
6. Synthesizing—creating new ideas or extending/revising understanding based on engagement with texts or mathematic observations/investigations
7. Monitoring meaning—thinking about the degree of understanding and taking steps to improve understanding when necessary

(Adapted from Keene and Zimmermann 2007)

Anyone who works closely with young learners knows that when students feel competent and successful with a given activity, their interest in it soars. Reading teachers are aware that confident readers tend to read more, and the more they read, the more their skill increases. As a result, their confidence increases, prompting them to read even more. Unfortunately, students who lack confidence are likely to avoid reading. Once this occurs, the ability gap between the confident readers and those who lack confidence grows over time. Therefore, teaching all students how to become confident readers may prevent, or at least minimize, achievement gaps. And, according to Brassell and Rasinski (2008), “To become confident readers who readily comprehend, students need to have comprehension strategies. Teachers need to teach them these strategies.”



Similarly, students who are confident of their mathematical abilities are much more willing to tackle problems, communicate mathematically with others, and think critically about math-related ideas. And, the more they engage in mathematical activities, the more their mathematical skills improve. Just as having a toolbox of comprehension strategies for reading gives students confidence and improves their ability to read, these tools, when adapted for mathematics, have the same effect. Teachers can help their students recognize the interdisciplinary nature of the reading strategies they are already using and encourage them to use the same strategies to improve their understanding of mathematics.

It is interesting to note that Pólya (1957) in his text *How to Solve It*, listed four steps, the first of which was “understand the problem.” In more recent instructional materials, the first step has been revised and simplified to “understand the question” (O’Connell 2007). Students may understand the question, but still be stymied in their problem-solving efforts because they struggle with understanding the entire context of the problem. The comprehension strategies addressed in this book are powerful approaches that students can apply to help them understand these problems.

This book focuses on each of the comprehension strategies suggested by Keene and Zimmermann. In addition, Chapter 2 explores the importance of helping students increase their understanding of and ability to use mathematical vocabulary accurately, and offers suggestions for promoting vocabulary development in relevant mathematical contexts. Taking the literacy/mathematics connection even further, each chapter offers suggestions for incorporating children’s literature into math lessons. The infusion of these texts piques student interest, makes the relevance of mathematics in daily life apparent, and creates a bridge between reading and mathematics.

Teaching Students to Make Mathematical Connections

McGregor (2007) writes about making connections by describing the magic of a spider web:



“Spider webs are...magical. I gaze at them and think about the time and genius it took to create such masterpieces, works of art that go mainly unnoticed. What fascinates me is that these almost invisible connections link seemingly unrelated objects together. By early autumn in Ohio, you’ll discover that almost everything outside is webbed together if you stop to notice.”

Few of us notice or appreciate the beauty of spider webs. They tend to be ignored or brushed away until someone stops and points them out.

Too often, the wealth of mathematical connections students accumulate suffer the same fate as spider webs. In their haste to learn new mathematical material, many fail to slow down and draw upon rich connections to their previous experiences. But with scaffolding to support thoughtful reflection, students are able to call upon their background knowledge to make math-to-self, math-to-math, and math-to-world connections as they construct mathematical meaning and wrestle with mathematical problems.

Modeling and Think-Alouds

Teach students to understand and use comprehension strategies through modeling and think-alouds. It sounds easy enough, but to be done well it requires forethought and preparation. There must be a clear focus. To ensure that think-alouds sound genuine and conversational, Miller (2002) suggests:

- **Proper planning to prevent poor performance.** Although most teachers are capable of “winging it” if necessary, it is difficult to model the thinking involved when making mathematical connections without prior reflection. Before presenting a mathematical concept or problem, consider what mathematical connections can be made that generate interest and clarify thinking.

Construct a framework for thinking that students can emulate as they work with mathematical ideas. Will the connections be math-to-self, math-to-math, or math-to-world? Why is a connection of value? When will you describe the connection?

- **Authenticity matters.** Students love to hear about the experiences of their teachers. When studying area, if you are a pet owner, model a math-to-self connection about planning a dog run: How much area does the dog need? Is the size of the dog important to consider? That connection is relevant to the teacher and is, therefore, authentic.

Or, share how area relates to a hobby like scrapbooking. A math-to-math connection might be modeled by explaining how to use a ruler to accurately measure the length of a side of a shape to determine how much paper is required to create the shape with a given area. Thinking aloud reminds students that linear measure is mathematically linked to the measurement of area.

Make a math-to-world connections through newspaper articles, such as a description of the number of square miles impacted during a flood. The more specific and genuine the connections, the more impact they have on student learning.

- **Use precise language.** As you model and think aloud, use precise and concise mathematical language, particularly for mathematical connections, and be consistent with the use of the terms.

Use the same set of sentence stems that leads to making connections, so that students become familiar with them and apply them automatically as they make connections. After several think-alouds, challenge the class to create a list of sentence stems that connect math to their prior knowledge:

- I remember that...
- This is just like when...
- I know that...
- This reminds me of...
- That is similar to...