

Guided
MATH

Daily
Math *Stretches*

Building Conceptual Understanding

Levels 6-8

Authors


Laney Sammons and Pamela Dase



SHELL EDUCATION

Up and Down *Stretch*




Standard

-  Adds, subtracts, multiplies, and divides integers, and rational numbers

Overview

Students investigate the rules for multiplying integers.

Materials

-  chart paper
-  markers
-  scientific or graphing calculator

Warming Up for the Stretch

Prior to doing this Stretch, students should have learned about integers and how they are ordered. They should have performed operations with negative numbers with and without calculators.







Up and Down *Stretch* (cont.)

Stretch Procedure

1. Prepare a class chart with the problems included for investigation of multiplying positive and negative integers. (See sample on page 43.)
2. Display the chart as students enter the room.
3. Have the students choose a problem and write the answer in the appropriate place on the chart paper.
4. When all students have finished entering their answers on the chart, meet as a class in a Math Huddle. You can use the questions below to guide this discussion.

Suggested Questions for Informal Assessment: Math Huddle

Level of
Teacher
Support

-  What do you notice about how the products change in problems 1–9? (Students should notice as the product decreases by 4, the second factor decreases by 1.) Why does this occur? What does this tell you about multiplying a positive number by a negative number?
-  How do the products change in problems 10–18? Why do you think the answers to 10–14 are the same, but in reverse order, as the answers to 6–9? Read further down the chart; what is happening to the value of each product? What does this tell you about multiplying a negative number by a negative number?
-  How are problems 19–27 different from the others? Which problems have positive answers? How many negative numbers were used as factors in each of these problems? Was there an even or odd number of negative factors?
-  Which problems have negative answers? How many negative numbers were used as factors in each of these problems? Was there an even or odd number of negative factors?
-  What is the rule for multiplying several negative numbers? Do these same rules work if there are just two factors?
-  Do you think this rule would work if the problem had a mixture of positive and negative numbers? Why do you think it would or would not work? Think of a problem to check on your calculator.

Up and Down *Stretch* (cont.)



What It Looks Like: Stretch Snapshot

This Stretch Snapshot helps students discover the patterns that explain the basic rules for determining the sign of the product when multiplying two integers. Generating the rules for multiplying two integers should be easy and will help them to recognize that, given more than two factors, the product is negative if there are an odd number of negative integers, but positive if there are an even number of negative integers. Using the associative property to group numbers is sometimes useful. For example, $2 \times -3 \times -4 \times 5 \times -6$ can be multiplied as $2 \times -3 = -6$, then $-6 \times -4 = 24$, then $24 \times 5 = 120$, and $120 \times -6 = -720$. This shows that multiplication is a binary operation, and each negative sign determines the sign of one of the multiplication problems.

In this Math Huddle, the teacher feels that Elena does not completely understand how to determine the sign of a multiplication problem that contains more than two factors.

Teacher: *Elena, think about the products we found for the problems on the chart. From the patterns we discovered, how could you determine whether the answer to a multiplication problem is positive or negative?*

Elena: *If there is one negative, the answer is negative. If there are two negatives, the answer is positive.*

Teacher: *Does everyone agree?*

Trent: *I do. Look at the chart. Every time we have just one negative factor, the product is negative. When there are two negative factors, the products are always positive—except for problems like Elena’s problem, $2 \times -3 \times -4$.*

Teacher: *Elena, can you find two problems where Trent’s rule works?*

Elena: *Yes, $2 \times -3 = -6$ because there is one negative. And $-2 \times -3 = 6$ because there are two negatives.*

Teacher: *Does everyone agree with Elena? (Students nod.) I think you understand how to multiply two numbers together. What do you do if there are more than two factors in the multiplication problem? Turn to your elbow partner and share what you think. (The teacher gives students time to share their ideas.) Now, Elena, what do you think?*

Elena: *You count and see if it’s even or odd.*

Elena: *You count the numbers and see if it’s even or odd.*

Teacher: *Are you saying that we count how many numbers are being multiplied, the total number of factors? Let’s look at your problem on the chart: $2 \times -3 \times -4$.*

Up and Down *Stretch* (cont.)



What It Looks Like: Stretch Snapshot (cont.)

- Elena:** *I think that $2 \times -3 \times -4$ should be -24 because there are three numbers.*
- Trent:** *I don't think it will be negative.*
- Teacher:** *Why not?*
- Trent:** *Well, I remember that you can arrange the numbers in a multiplication problem in any order that you want. If you multiplied 2×-3 first, you get -6 because there is one negative.*
- Teacher:** *Okay. Where do you go from there?*
- Trent:** *Now I would multiply that answer, -6 , times -4 .*
- Elena:** *Hey, that's 24 , because there are two negatives.*
- Teacher:** *Elena, what do you think? Look at the original problem. How many negatives are there?*
- Elena:** *Two. Oh, it's the number of negatives we multiply—not all the numbers that are multiplied!*
- Teacher:** *Let's try one. Think about $2 \times -3 \times 4 \times 2 \times -3 \times -4$. Multiply the numbers out two at a time and then multiply your answers together.*
- Elena:** *2×-3 is -6 ; 4×2 is 8 , and -3×-4 is 12 ; -6×8 is -48 ; and -48×12 is -576 .*
- Teacher:** *Why do you think the product is negative?*
- Elena:** *There are three negatives. Three is an odd number.*
- Teacher:** *What if you changed one of the positive numbers to a negative number? How many negatives would there be?*
- Elena:** *There would be four negatives.*
- Teacher:** *Would the answer be positive or negative?*
- Elena:** *Positive.*
- Teacher:** *So, mathematicians, anytime that you are multiplying a number of factors—some of which are positive and some of which are negative—what did we discover? How can we know if the product will be positive or negative?*
- Elena:** *Count the negatives. An odd number of negatives means that answer is negative. An even number of negatives means that the answer is positive.*
- Teacher:** *Good job, Elena. That's exactly right. Always remember to use that rule when you are multiplying positive and negative factors.*

Up and Down *Stretch* (cont.)

Sample Chart

Choose a problem below and solve it with and without a calculator. If the answers are different, put both answers on the chart. Add your initials.

1. $4 \times 4 = 16$ BG	10. $-4 \times 4 = -16$ NB
2. $4 \times 3 = 12$ KM	11. $-4 \times 3 = -12$ HM
3. $4 \times 2 = 8$ LI	12. $-4 \times 2 = -8$ JB
4. $4 \times 1 = 4$ CH	13. $-4 \times 1 = -4$ JL
5. $4 \times 0 = 0$ MK	14. $-4 \times 0 = 0$ CJ
6. $4 \times -1 = -4$ ED	15. $-4 \times -1 = 4$ RT
7. $4 \times -2 = -8$ SP	16. $-4 \times -2 = 8$ MT
8. $4 \times -3 = -12$ QW	17. $-4 \times -3 = 12$ DM
9. $4 \times -4 = -16$ ZS	18. $-4 \times -4 = 16$ DB
19. $2 \times 3 \times 4$ has <u>0</u> negative signs and the answer is <u>24</u> GG.	
20. $2 \times 3 \times -4$ has <u>1</u> negative signs and the answer is <u>-24</u> NY.	
21. $2 \times -3 \times -4$ has <u>2</u> negative signs and the answer is <u>-24</u> ER.	
22. $-2 \times -3 \times -4$ has <u>3</u> negative signs and the answer is <u>-24</u> GG.	
23. $-1 \times -2 \times -3 \times -4$ has <u>4</u> negative signs and the answer is <u>24</u> BM.	
24. $-1 \times -2 \times -3 \times -4 \times -5$ has <u>5</u> negative signs and the answer is <u>-120</u> SH.	
25. $-1 \times -2 \times -3 \times -4 \times -5 \times -6$ has <u>6</u> negative signs and the answer is <u>720</u> SE.	
26. $-1 \times -2 \times -3 \times -4 \times -5 \times -6 \times -7$ has <u>7</u> negative signs and the answer is <u>-5040</u> GG.	

How to Use This Product



Number and Operations: Stretches

What Comes First? *Stretch*

Standards

- Understands the basic meaning of place value
- Understands the relative magnitude and relationships among whole numbers, fractions, decimals, and mixed numbers
- Uses explanations of the methods and reasoning behind the problem solution to determine reasonableness of and to verify results with respect to the original problem

Overview

With the What Comes First? stretch, students compare and order whole numbers, fractions and decimals. These numbers are determined by the teacher. Rigor can be increased or decreased based on students' conceptual understanding.

Materials

- What Comes First? chart (the set of numbers selected by the teacher is written in the first column of the chart; there should be one number for each student in the class)
- markers
- a place value chart may be displayed in the room for student reference

Warming Up for the Stretch

Problems similar to the What Comes First? task should be introduced during calculator-based instruction. Teachers should model comparing and ordering whole numbers, decimals, and fractions and invite student feedback as they solve each problem. Students may begin with teacher demonstrations of finding sections of 10 by 10 grids as a symbolic representation of numbers. Then the teacher thinks aloud as he or she places the numbers in order from largest to smallest and then from smallest to largest. Throughout the work, the teacher increases the difficulty by moving from whole numbers without decimals to whole numbers with decimals, from like to unlike fractions, and then to mixed numbers. To informally assess student understanding, place students in small groups and assign each student a decimal or fraction. The group then lines up in order from smallest to largest or largest to smallest.

© Shell Education #50786—Math Stretches 39

Each section opens with a list of the standards that are represented by the activity, followed by an overview of the stretch.

A list of any materials and necessary prerequisite instruction (Warming Up for the Stretch) are included to help the teacher prepare the classroom and the students for the activity, minimizing the need for teacher assistance and allowing the students to have as much independence as possible to complete the task.

A simple, step-by-step procedure directs the teacher in how to conduct the Math Stretch. Included in this section are suggestions for extending the stretch for further mathematical exploration, as well as modifications for students who are nonreaders.

The Math Huddle section suggests questions for informal assessment that a teacher can ask to provide varying levels of support and to facilitate a gradual release of responsibility (see more about Inquiry-based Learning on page 33).

Number and Operations: Stretches

What Comes First? *Stretch* (cont.)

Stretch Procedure

- Display the What Comes First? chart on the board or the interactive whiteboard so students see it as they enter the classroom. The first student to work on the stretch looks at the numbers in the first column and determines which has the least value. He or she then places that number in the first cell of the second column and adds his or her initials. Subsequent students decide which number from the first column to place next in the second column to keep the order from least to greatest.
- Each student records a number in the second column with his or her initials. If a student thinks a number has been incorrectly added to the second column, he or she should question the student who added that number. Only the student who recorded a number may change it. This communication will provide practice for students as they share the mathematical justification for their ideas.
- Once all students have completed the stretch, call the students together for Math Huddle. The questions below may be used to lead the discussion.

Suggested Questions for Informal Assessment: Math Huddle

Level of Teacher Support

- Look at the completed set of numbers. Are there any numbers that you think are in the wrong order? If so, why do you think that?
- How did you decide that the first number listed had the least value? How did you decide which number came next? Which number came last?
- Think of other ways to compare these numbers. Can you round them? Is it helpful to read them in word form? Could you use graph paper to compare the value of the numbers?
- Why is it important to understand place value when comparing and ordering numbers?
- Can you think of a time when you needed to compare numbers? Have you ever been in a grocery store with two different brands of the same kind of candy? One brand costs \$1.50, the other costs \$1.99. Which candy would you buy if you wanted to spend less money? Why? How much would you save?
- Can you think of any other times when it may be important to determine the order of numbers from least to greatest or from greatest to least?

40 #50786—Math Stretches © Shell Education

How to Use This Product *(cont.)*



Number and Operations Stretches

What Comes First? *Stretch* (cont.)

What It Looks Like: Stretch Snapshot

This Stretch Snapshot offers teachers an easy way to assess their students' understanding of place value and their ability to compare and order numbers. However, this assessment may not be completely accurate. If there is a mistake in the order of numbers, teachers should determine where the error was made and confer with the student to assess whether this was a calculation error or an error in the conceptual understanding of place value.

The class in this Stretch Snapshot has been focusing on comparing the value of decimals to the thousandths place. The discussion revolves around the class's decision to place 4.006 before 4.06 in the set of ordered numbers going from least to greatest. Although the ordering of the decimals is correct, the teacher asks students to justify their responses using mathematically correct vocabulary. This allows the teacher to informally assess students' understanding and reasoning skills.

Teacher: I see Michael placed 4.006 here on the chart and, Kallee, you placed 4.06 next. Explain why you think 4.06 is greater than 4.006.

Kallee: I looked at the two decimal numbers and saw that the hundredths place in one was a zero and in the other one it was a 6.

Teacher: Michael, do you agree with Kallee?

Michael: I think it's called the hundredths place.

Teacher: Yes, it is. Class, remember that when working with decimals, the place value to the right of the decimal is expressed by adding "-ths" to the end. Kallee, you were explaining your math reasoning?

Kallee: Oh, yeah! I saw that the zero in the hundredths place was smaller and that would make the decimal smaller than the number with a 6 in the hundredths place. There were zeros in the tenth place of both numbers.

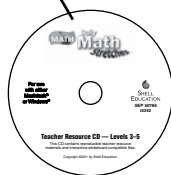
Teacher: But there is a 6 right there, Kallee. (Teacher points to the 6 in the thousandths place in the number 4.006.) Why doesn't that make it greater? There are more digits in this number. (Kallee looks puzzled.)

© Shell Education #50786—Math Stretches 41

The Math Stretch concludes by providing the teacher with a model of how the Math Huddle may look in an actual classroom setting. This Stretch Snapshot illustrates the kinds of conversations teachers can have with their students, demonstrating how to extend students' thinking or uncover the sources of students' confusion about a concept. These dialogues model guided inquiry, in which the teacher facilitates the conversation, so that students can make connections and discover underlying themes on their own.

A sample of the chart or poster created during each stretch is also provided. The specific information shown in each sample mirrors the content of the Stretch Snapshot.

Templates of each chart or poster are available on the Teacher Resource CD, as well as electronic formats using interactive whiteboard technology.



Number and Operations Stretches

What Comes First? *Stretch* (cont.)

Sample Chart

What Comes First?

Using the numbers in the left column, find the number that comes next so that the numbers will be in order from the least to the greatest. Place your choice in the second column along with your initials. Cross out the decimal in the first column that you chose. The first one has been done for you.

Decimal Set	Ordered Decimal Set
4.199	3.999
4.999	4.205 P.W.
4.791	4.06 K.S.
4.199	4.300 T.R.
0.999	4.321 P.L.
4.999	4.321 L.C.
4.789	4.254 C.D.
4.787	4.300 R.W.
4.099	4.387 S.G.
4.291	4.389 W.L.
4.788	4.398 N.O.
4.191	4.399 D.R.
4.791	4.209 J.M.
4.789	4.209 S.L.
4.799	4.269 M.L.
4.791	4.70 D.H.
4.703	4.703 A.W.
4.703	4.705 A.T.
4.709	4.741 W.
4.703	4.765 P.J.

© Shell Education #50786—Math Stretches 43

Introduction *(cont.)*



Guided Math: A Flexible Framework for Mathematics Instruction

As teachers, we struggled to find practical ways to incorporate best practices into my mathematics instruction. Gradually, I developed a model that allowed me to offer every student an opportunity to develop his or her mathematical skills at increasingly challenging levels of difficulty. My ultimate goal was to help each gain the ability to function independently in the world of mathematics. The Guided Math framework I used is designed to support mathematical literacy by mirroring many of the same techniques applied to teaching literacy for many years.

Guided Math is broadly defined as a flexible instructional framework that enables teachers to promote the deep mathematical understanding and computational fluency of their students by determining their unique needs and prescriptively addressing those needs through a combination of whole-class instruction, small-group instruction, math workshops, and conferences within a classroom environment supportive of numeracy.

The specific instructional components of this model include:

1. A Classroom Environment of Numeracy
2. Morning Math Warm-Ups and Calendar Board Activities
3. Whole-Class Instruction
4. Guided Math Instruction with Small Groups of Students
5. Math Workshops
6. Individual Conferences
7. An Ongoing System of Assessment

Used together, these components allow teachers to implement research-based best practices in their classrooms that support each student's mathematical learning according to his or her needs.

A Classroom Environment of Numeracy

Environments rich in mathematical opportunities are essential if students are to develop a thorough understanding of mathematics. When students begin to recognize how numbers and problem solving affect their everyday lives, mathematics becomes more meaningful to them. Because learning is both a social and constructive process, children learn best through active engagement in authentic tasks that offer opportunities to use and extend their number senses.

Table of Contents



Acknowledgements	4	Algebra and Algebraic Thinking Stretches	
Introduction	5	Any Way You Look at It Stretch	64
Promoting Mathematical Literacy for the 21st Century with Math Stretches	5	At a Snail's Pace Stretch	69
Mathematical Literacy	5	Back at You Stretch	74
Current Mathematics Instructional Methods	5	Playing Catch-Up Stretch	79
Researched-Based Best Practices for Teaching Mathematics	6	Work Through It Stretch	84
Making Connections	6	Geometry Stretches	
Mathematical Communication	8	Across the Transversal Stretch	89
Vocabulary for Mathematics	9	It's Greek to Me Stretch	94
Repetition of Mathematical Learning	10	Triangulation Stretch	99
Guided Math: A Flexible Framework for Mathematics Instruction	13	Magic Triangles Stretch	104
Using Math Stretches to Promote Mathematical Literacy	20	Polygons by Degree Stretch	109
Planning Math Stretches	20	Measurement Stretches	
Preparing for the Math Stretch	20	"Per" fectly Stretch	114
Teaching Procedures for Math Stretches	23	Precisely Stretch	119
Math Huddles to Discuss the Math Stretch	25	How Big? Stretch	124
Types of Math Stretches	27	It Grows on You Stretch	129
How to Use This Product	31	Whatever Works Stretch	134
Correlation to Mathematics Standards	36	Data Analysis and Probability Stretches	
Number and Operations Stretches	39	Box It Up Stretch	139
Up and Down Stretch	39	Heads or Tails Stretch	144
Sift Them Out Stretch	44	Odds On Stretch	149
Order of Operations Stretch	49	Pizza Pie Stretch	154
It's Scientific Stretch	54	Right Down the Middle Stretch	159
Exponential Shortcuts Stretch	59	Across-the-Disciplines Stretches	
		How Did My Family Use Math Recently? Stretch	164
		Know and Want to Know Stretch	169
		___Makes Me Think Of . . . Stretch	174
		Numbers in the News Stretch	179
		We Need Mathematics Stretch	184
		References	189
		Contents of the Teacher Resource CD	191