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Think It, Show It Mathematics

Strategies for Explaining Thinking

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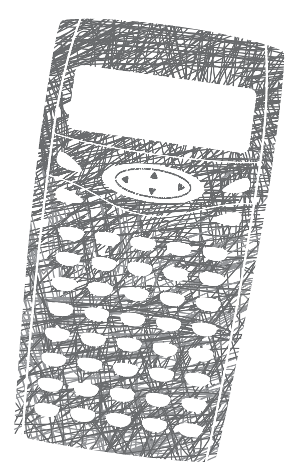


Introduction

Although I was unaware of it at the time, the catalyst for the development of the activities and strategies described in this book dates back to the fall of 1999. In my home state of Colorado, that time period saw the inception of the math assessment component of the state-mandated yearly *Colorado Student Assessment Program.* Colorado public school fifth graders were tested, for the first time, in mathematics. There was much apprehension about their students' performance on the fall assessment.

When the scores were released, the results were more disappointing than had been anticipated. Fifty percent of Colorado's fifth graders had not scored as proficient. The newspaper headlines left nothing to the public's imagination. Three of the largest newspapers in the state ran these front-page proclamations the day after the results were released:

> *Half of 5th-Graders Fail Math Test*
> Rocky Mountain News, March 3, 2000
>
> *The Math Challenge: More Than Half in Test Fall Short*
> Denver Post, March 3, 2000
>
> *Half of Fifth Graders Mastering Math*
> Pueblo Chieftain, March 3, 2000

Unfortunately, that spring, the Colorado eighth graders didn't fare much better. When districts across the state started analyzing the data from the tests, they identified a persistent weakness with the written or "constructed responses" required of students. Our fifth and eighth graders demonstrated significant difficulty in communicating problem-solving procedures and their mathematical reasoning in written form.

What we saw in Colorado was not unique to students in the "Mile High State." It was mirrored across the country. Since then, more evidence of our students not only struggling with math literacy but also lagging behind other nations has been documented. The National Mathematics Advisory Panel's final report, *Foundations for Success,* cited a 2007 study that revealed that 15-year-olds in the United States ranked 25th among their peers in 30 developed nations in math literacy and problem solving.

Simply put, our students struggle with communicating their mathematical reasoning in a verbal or written form—not to mention a resistance when asked to do so. *If I got the answer correct, why do I have to explain how I got it?* is a typical response. As a result, a number of schools began to ask me to work with their students on the writing required in both the language arts and the written portions of the math test. And now, with the advent of the *Common Core State Standards,* I continue to refine the work that I do with teachers and students in order to meet those objectives. *Think It, Show It Mathematics: Strategies for Explaining Thinking*, along with its materials, is a result of my work and research with teachers and students in an effort to help all students better demonstrate their mathematical understandings through writing and discussion.

Through this work, I have found ways to merge practical and dependable writing strategies with ongoing day-to-day mathematics instruction. For example, students use “framed paragraphs” to facilitate formulas for a written explanation of the solving of a word problem and thereby learn how writing in mathematics needs to be read and sound. The language of these “mathematical framed paragraphs” later functions as mental templates that students can fall back on, as needed, as they mature and begin working with more challenging problems. By becoming proficient with these types of paragraphs, students can internalize the sound and structure of a basic math narrative.

Below is a third-grade written explanation sample. For readability purposes, students’ spelling errors have been corrected throughout the book.

> To solve the problem, I first added the 9 absent boys and the 3 absent girls and found 12 students. After that, I subtracted the 12 students from the 23 students that were in class and found 11 students. Therefore, I know Alicia’s class had only 11 students left.
>
> 3rd Grade Student

Another example is using the method of writing a mathematical procedural text employing “what” and “why” statements: operations and corresponding reasons. “What” statements are statements of what mathematical operations students need to do to solve the problem, and “why” statements are statements explaining why they need to use these operations with specific references to contextual details found in the stated problem. Included with these explanations are “why” words *(since* and *because)* along with a handful of good procedural transition or sequence words *(to start with, first, then, after that, second, etc.),* and the perfect wording to start a concluding sentence *(Therefore, I know):*

Chapter 2

One- and Two-Step Problems

Problem solving in mathematics forms a foundation for student success and "is not only a goal of learning mathematics but also a major means of doing so" (NCTM 2000, 4). According to the *Common Core State Standards*, students must be able to apply mathematics to solve problems that occur in everyday life as well as explain their thinking and communicate their solution (2010). By solving mathematical problems, students experience opportunities to think mathematically, persevere in solutions, develop curiosity and confidence, and make connections to mathematics and the real world (NCTM 2000). As suggested in the opening chapter, problem solving allows teachers to build mathematically empowered students and requires students to think critically and creatively in order to arrive at solutions.

This level of thinking, problem solving, and mathematical competence is even more critical today with an increasing number of students using calculators. As essential tools in our classrooms and lives, calculators serve us by giving us right answers, but reliance on them can allow students to circumvent an understanding of the mathematical principles involved with a given situation.

I am reminded of my attempt to buy a sweater at a department store awhile back. It was a $50 sweater reduced by 20 percent. A great deal! So I picked up the sweater and headed to a counter. Unfortunately, the clerk there had misplaced her calculator and was flustered. "You'll need to go to the other counter," she apologized. I looked and saw a line of six or seven people. Not wanting to deal with a long wait, I simply told the clerk that 20 percent of $50.00 is $10.00 and if we subtracted that from the $50.00, the sweater would cost $40.00. "So just key $40.00 into your register and figure the tax and I'll be on my way," I suggested. But the clerk still insisted that she needed her calculator. "It's not that I don't trust you," she confessed, "but I always use my calculator." Finally, she had an idea and picked up her telephone, called another clerk, and explained, word-for-word, exactly what keys to press on her calculator, and they found—lo and behold—that with the 20 percent discount, the sweater would only cost $40.00.

I am not being terribly critical of the clerk (trust me—I rely heavily on my calculator when averaging grades, balancing my check book, and doing taxes!). It does, however, illustrate how important it is for students to go beyond the computations afforded by a calculator to understanding the mathematical processes and principles underlying what they key into their calculators. In the case of my department store clerk, it was to understand the function of percentages in determining the price of a discounted sweater.

Another case in point returns us to the fourth-grade students who were working with the problem of Todd and the carpeting of his game room that I discussed in the first chapter. Earlier in the year, these same students had been given activity sheets on which they practiced calculating perimeters and areas that were presented pictorially. But later in the year some of them were baffled when the two mathematical tasks were placed side-by-side in the real-life context of Todd and the square feet of carpet required for his game room. Our students need to become skilled in addressing and understanding word problems as they might be encountered in everyday life.

Addressing Word Problems

We start teaching students how to explain their mathematical thinking by showing them how to first systematically address word problems. The process follows these steps:

Read → Decide → Estimate → Work → Explain

1. **Read the Problem**
   - What is happening in the problem?
   - What do I know?
   - What don't I know?
   - What is the problem asking me to find out?

2. **Decide**
   - What operation(s) will I need to do to solve the problem?
   - What strategy will I use to solve the problem?

3. **Make an Estimation**
   - What is a reasonable answer?

4. **Work the Problem**
   - Check my work.

5. **Explain My Math Thinking in Writing**

The first step with any word problem is to establish what is happening. The *Common Core State Standards for Mathematics* describes mathematically proficient students as ones who "start by explaining to themselves the meaning of a problem" (2010, 6). Students should be regularly asked to explain in their own words what is happening in a given problem. A fun yet effective way to do this is to have students reframe what is happening as if they were telling it as a story by starting with the words *once upon a time*. Here is a student example based on the following word problem:

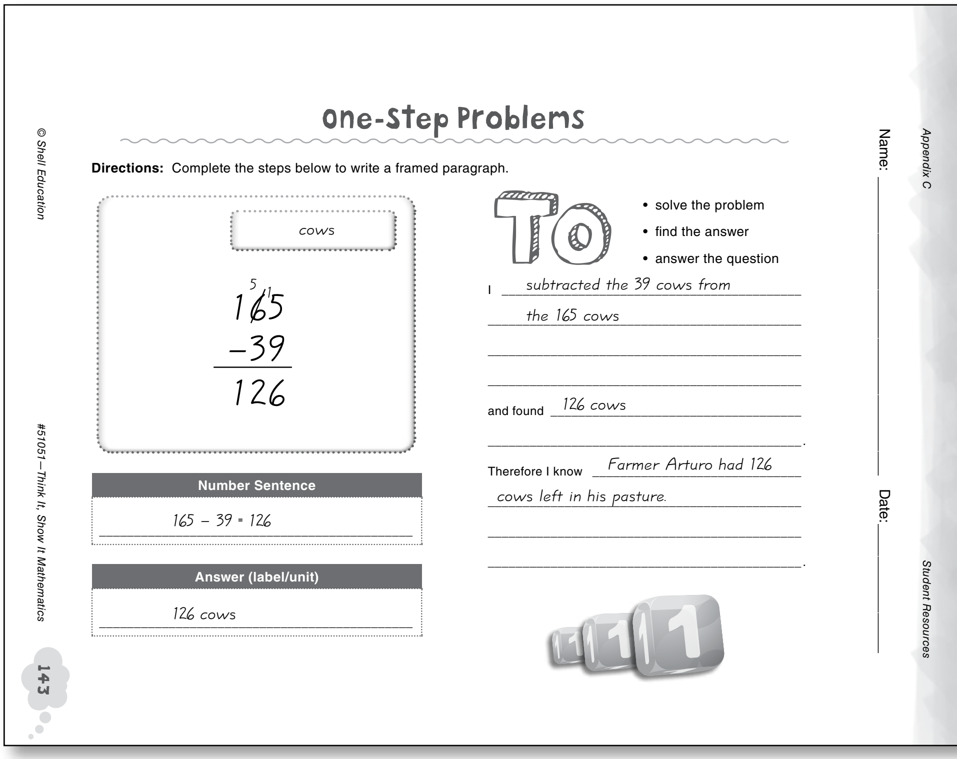
> Farmer Arturo had 165 cows in his pasture. One night, some pranksters opened his gate, and 39 cows wandered out into his neighbor's pasture. How many cows does Farmer Arturo have left in his own pasture? Explain how you found your solution.

> Once upon a time, there was a farmer named Farmer Arturo who had 165 cows in his pasture. But one day, some guys came in and let 39 of his cows out of his pasture. Farmer Arturo was so sad! He wanted to know how many of his cows he had left.
>
> 3rd Grade Student

Students are then asked to share their "stories" with their learning partners or to share and listen to one another in cooperative learning groups. Students are also encouraged to visualize the problem's situation and make a drawing or sketch to represent the problem situation. This is important because students often want to read problems as quickly as they can and jump immediately into doing the calculation. The desired result is "I'm the first one done!" being loudly announced as their arms and pencils are launched straight up into the air. This "being-done-first" mindset, of course, is common among many students. It is true that in the early grades, many students often are able to correctly solve some problems without careful and analytical reading. However, as they progress into more difficult and multifaceted mathematics, this assuredly will not continue to be the case. Students need to develop a mathematical "habit of thought" when working with word problems by always explaining to themselves what is happening in the problem.

Fundamental in the careful reading of a word problem in order to solve it is the determination of the essential information. They are the "givens" or "what do I know?" Along with "what do I know," students need to address "what don't I know." Determining this often involves the identification of the words.

Figure 2.6 One-Step Problems Student Sample



Appendix C

Name: ______

One-Step Problems

Directions: Complete the steps below to write a framed paragraph.

cows

$$\begin{array}{r} 1\overset{5}{\cancel{6}}{}^{1}5 \\ -39 \\ \hline 126 \end{array}$$

Number Sentence

165 – 39 = 126

Answer (label/unit)

126 cows

To

- solve the problem
- find the answer
- answer the question

I subtracted the 39 cows from the 165 cows

and found 126 cows.

Therefore I know Farmer Arturo had 126 cows left in his pasture.

Date: ______

Student Resources

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To solve the problem, I subtracted the 39 cows from the 165 cows and found 126 cows. Therefore, I know Farmer Arturo had 126 cows left in his pasture.

3rd Grade Student

Try It!

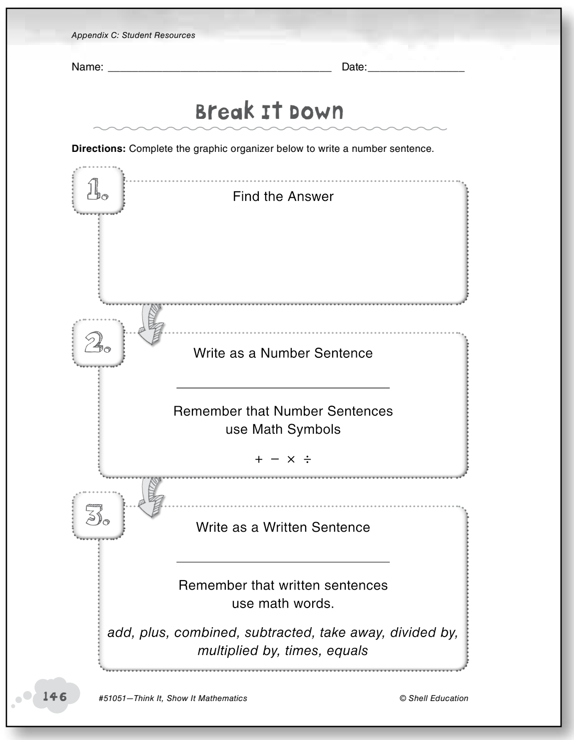
After using the *One-Step Problems* activity sheet you may want to extend your students' learning by having them take their number sentence (165 – 39 = 126) and write and illustrate their own word problems.

There are 165 second graders at Emerson Elementary School, but one day 39 of them were sick and didn't come to school. How many students were left at school that day?

3rd Grade Student

There may be some students who are not ready to work independently with the *One-Step Problems* activity sheet. They may struggle not only with the computation, but also with writing the number sentences and then transferring them into a written sentence. Figure 2.7 displays an activity sheet that can be used to give these students practice with this sequence. To use this activity sheet, students must first be provided a problem. The problem is to be worked in the first box using drawings, number strips, or manipulatives—whatever is most helpful for students—to find the answer. In box two, students record the operation as a number sentence. Finally, students take their number sentence and write it as a written sentence in box three, where they have the spelling of the words they need.

Figure 2.7 Break It Down



Appendix C: Student Resources

Name: ______________________ Date: ____________

Break It Down

Directions: Complete the graphic organizer below to write a number sentence.

1. Find the Answer

2. Write as a Number Sentence

____

Remember that Number Sentences use Math Symbols

+ − × ÷

3. Write as a Written Sentence

____

Remember that written sentences use math words.

add, plus, combined, subtracted, take away, divided by, multiplied by, times, equals

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Here is another example of a one-step problem solved by using the *One-Step Problems* activity sheet.

> Chris wants to get cool new headbands for his three buddies to use when they play basketball at recess. He knows that each headband will cost $4.36, including tax. How much money will Chris need to buy the headbands?

Add-an-Attribute Poem

A simple, yet highly effective pattern that students can use when researching and thinking about the attributes of a certain concept in math is called an *Add-an-Attribute poem*. This type of activity helps as they are learning to read and locate relevant information. *Add-an-Attribute* poems simply start with *A* or *An* as the first line, and the noun being described is added as the second line. Each successive line repeats the previous line and adds an attribute, hence, an *Add-an-Attribute* poem. This type of poem can be used to describe any mathematical concepts such as geometric figures, factoring, area, volume, measurements, or fractions.

One way this was enacted in the classroom was using geometric shapes as the subjects of the poem. Students were provided an *Add-an-Attribute Planning Sheet* (Figure 4.3). On their planning sheets, students included the name of their shape and recorded their research as they studied them:

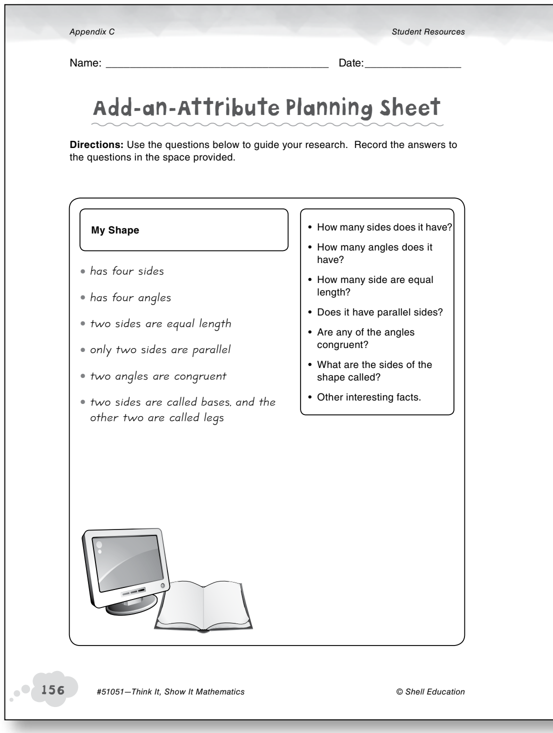
- How many sides does it have?
- How many angles does it have?
- How many sides are equal length?
- Does it have any parallel sides?
- Are any of the angles congruent?
- What are the sides of the shape called?
- Other interesting facts?

Students filled out the sheet with a bulleted jot list:

- has four sides
- has four angles
- two sides are equal in length
- only two sides are parallel
- two angles are congruent
- two sides are called bases, and the other two are called legs

Using the information from their planning sheet, students then composed their poems. Finally, students had an opportunity to take their poems to other classrooms and, without giving away their last line, have other students guess what shape each student had researched.

Figure 4.3 Sample Add-an-Attribute Poem Planning Sheet



Appendix C — Student Resources

Name: ______________________________ Date: ______________

Add-an-Attribute Planning Sheet

Directions: Use the questions below to guide your research. Record the answers to the questions in the space provided.

My Shape

- has four sides
- has four angles
- two sides are equal length
- only two sides are parallel
- two angles are congruent
- two sides are called bases, and the other two are called legs

- How many sides does it have?
- How many angles does it have?
- How many side are equal length?
- Does it have parallel sides?
- Are any of the angles congruent?
- What are the sides of the shape called?
- Other interesting facts.

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Here is a student's poem best read aloud to get a feel for its poetic nature with the repetition.

Sample Student Add-an-Attribute Poem

A

A shape

A geometric shape

A four-sided geometric shape

A four-sided quadrilateral geometric shape

A four-sided quadrilateral geometric shape with a pair of opposite parallel sides

A four-sided quadrilateral geometric shape with a pair of opposite parallel sides called bases

A four-sided quadrilateral geometric shape with a pair of opposite parallel sides called bases that are parallel and whose other sides are called legs

(*an isosceles trapezoid*)

Here is a student sample using a pyramid:

Sample Student Add-an-Attribute Poem

A

A shape

A geometric shape

A five-sided geometric shape

A five-sided geometric shape with a quadrilateral polygon for a base

A five-sided geometric shape with a quadrilateral polygon for a base and four faces

A five-sided geometric shape with a quadrilateral polygon for a base and four faces made of congruent triangles

What kind of shape am I?

(*a pyramid*)

Creatively bringing math writing into the writing class and making the most of the writing process helps students learn more about their math content and also supports their growth as writers. It helps them gain greater skill, control, and independence with their writing. Along with feeling more successful, it deepens students' understanding of the processes and functions of writing. They sense that they are becoming real math writers. Part of this emerging awareness of our "real writers" is their understanding of the different purposes and types of writing.

Writing Purposes and Different Types of Writing

To help students understand the different purposes and types of writing that are important in math, I start by having them close their eyes to imagine their "dream car".

> *If you had all the money you would ever need, what car would you have in your driveway?*

When they have fully visualized all of its details and are totally enthralled by their imagined dream car, I say:

> *Now, drop the engine out. So what do you have now?*

Acute and Obtuse Angles

The mentor text below is annotated to support you during instruction. The student version of this text is provided on the Digital Resource CD (mentortext2.pdf).

Two types of angles used in geometry are acute and obtuse angles [**topic and subjects**]. Although they are similar in many ways [**words used to compare**], there are, however [**transition word**] important differences between [**words used to compare**], acute and obtuse angles [**subjects and topic sentence**]. To begin with [**transition words**], both acute and obtuse angles have two lines or rays that share a common point called a vertex [**1st similarity**]. They are both measured in degrees [**2nd similarity**]. In addition [**transition word**], obtuse and acute angles are alike [**words used to compare**] because both can be used to create a triangle [**3rd similarity**]. Furthermore [**transition word**], an acute angle creates an acute triangle, and an obtuse angle makes up an obtuse triangle [**4th similarity**]. Similarly [**word used to compare**], both angles can be measured with a protractor and compass [**5th similarity**].

In contrast [**words used to compare**] the biggest difference between [**words used to compare**] acute and obtuse angles is their measurement. An acute angle will measure less than 90° [**1st difference**]. On the other hand [**words used to compare**], an obtuse angle measures between 90° and 180° [**2nd difference**]. This means that the opening of an obtuse angle is wider than the opening of an acute angle [**elaboration on 2nd difference**]. Finally [**transition word**], an acute angle has a measure or is smaller than a right angle, but an obtuse angle has a larger measure or is larger than a right angle[**3rd difference**].

Annotated Mentor Text: Comparative

Demonstrating topic, subjects, topic sentence, transition words, similarities and differences between subjects, elaboration, and words used to compare.

Name: ______________________________________ Date: __________________

Explanatory Text Planning Chart

Directions: Write your topic at the top of the activity sheet. Add details and additional information in the appropriate spaces below.

Topic

Information	**Information**
Details	**Details**